Equação Exponencial

$$x^{\sqrt{x}} = 2^{\sqrt{x}+12}$$

Com $t = \sqrt{x}$, temos que $t^2 = x$

Então: $(t^2)^t = 2^{t+12} = 2^t.2^{12} = 2^t.4^6$

Ou: $\frac{(t^2)^t}{2^t} = 4^6 \rightarrow \left(\frac{t^2}{2}\right)^t = \frac{4^7}{2^2} = \frac{4^8}{2^4} = \left(\frac{4^2}{2}\right)^4$

Resumindo: $\left(\frac{t^2}{2}\right)^t = \left(\frac{4^2}{2}\right)^4 \rightarrow t = 4$

Agora: $x = t^2 \rightarrow x = 4^2 \rightarrow \boldsymbol{x = 16}$

Outra maneira (numericamente):

$$(t^2)^t = 2^{t+12} \rightarrow t^{2t} = 2^{t+12}$$

$$2t.\ln(t) = (t+12).\ln(2)$$

$$t_{k+1} = \frac{(t_k+12).\ln(2)}{2\ln(t_k)}$$

Iniciando com $t_0 > 1$,

$$t_k \rightarrow 4$$

t	(t+12)ln(2) / 2.ln(t)
2	7
7	3,3839683
3,383968	4,3736362
4,373636	3,8456832
3,845683	4,0771299
4,07713	3,9646615
3,964661	4,016878
4,016878	3,9920942
3,992094	4,0037374
4,003737	3,9982409
3,998241	4,0008297
4,00083	3,999609

t	(t+12)ln(2) / 2.ln(t)
3,998241	4,00083
4,00083	3,999609
3,999609	4,000184
4,000184	3,999913
3,999913	4,000041
4,000041	3,999981
3,999981	4,000009
4,000009	3,999996
3,999996	4,000002
4,000002	3,999999
3,999999	4
4	4

$$\left(\frac{x}{5}\right)^{x} = 5^{25} \rightarrow x.\ln(x/5) = 25.\ln(5) \rightarrow x = \frac{25.\ln(5)}{\ln\left(\frac{x}{5}\right)}$$

Com "chute" inicial x_0 acima de 5, conseguimos uma sequência de valores:

$$x_{n+1} = \frac{25.\ln(5)}{\ln\left(\frac{x_n}{5}\right)}$$

x	25ln(5)
	ln(x/5)
6	220,6867
220,6867	10,6239
10,6239	53,38684
53,38684	16,99063
16,99063	32,89337
32,89337	21,35855
21,35855	27,71043
27,71043	23,49721
23,49721	26,00156
26,00156	24,40438
24,40438	25,38026
25,38026	24,76769
24,76769	25,14586
25,14586	24,90996
24,90996	25,05617
25,05617	24,96519
24,96519	25,02167
25,02167	24,98655
24,98655	25,00836
25,00836	24,99481