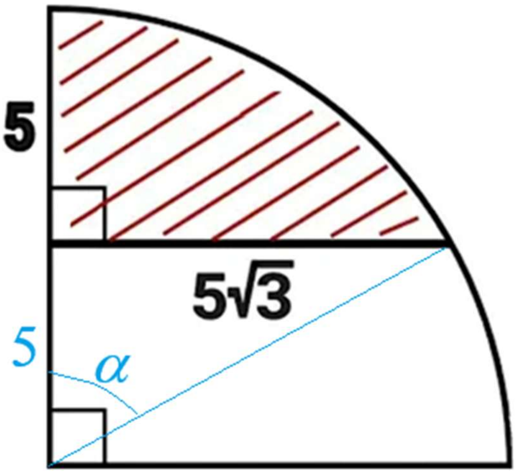
Calcular a área *hachurada*



$x^2 + y^2 = r^2$ (*)

Quando $x = 5\sqrt{3}$, $y = r - 5$

Substituindo em (*):

$(5\sqrt{3})^2 + (r - 5)^2 = r^2$

$75 + r^2 - 10r + 25 = r^2$

$10r = 100 \Rightarrow r = 10$

$\tan(\alpha) = \frac{5\sqrt{3}}{5} = \sqrt{3} \Rightarrow \alpha = 60^o$

A(◸) $= \frac{5\times5\sqrt{3}}{2} = \frac{25\sqrt{3}}{2}$ (triângulo)

A(◿) $= \frac{\pi.5^2}{12} = \frac{25\pi}{12}$ (parte do círculo = 30º)

A(◗) $= \frac{\pi.5^2}{4} = \frac{25}{4}$ (parte do círculo = 90º)

A(◺) $= \frac{25\pi}{4} - \frac{25\pi}{12} - \frac{25\sqrt{3}}{2} = \frac{25}{12}(3\pi - \pi - 6\sqrt{3}) = \frac{25}{12}(2\pi - 6\sqrt{3}) = \mathbf{\frac{25}{6}(\pi - 3\sqrt{3})}$

Ainda: Esta área poderia ser calculada por uma integral definida da função

$x(y) = \sqrt{100 - y^2}$, tirada de $x^2 + y^2 = r^2 = 10^2 = 100$

A(◺) $= \int_5^{10} \sqrt{100 - y^2}\, dy = \frac{1}{2}[y\sqrt{100 - y^2} + 10^2 \text{ arcsen}(\frac{y}{10})]_5^{10} =$

$= \frac{1}{2}[10\sqrt{100 - 10^2} + 100 \text{ arcsen}(\frac{10}{10})] - \frac{1}{2}[5\sqrt{100 - 5^2} + 100 \text{ arcsen}(\frac{5}{10})] =$

$= \frac{1}{2}[100\ \frac{\pi}{2}] - \frac{1}{2}[5\sqrt{75} + 100\,\frac{\pi}{6}] = 25\frac{\pi}{2} - \frac{25}{2}\sqrt{3} - 25\,\frac{\pi}{3} = 25\frac{\pi}{6} - 25\,\frac{3}{6}\sqrt{3} = \mathbf{\frac{25}{6}(\pi - 3\sqrt{3})}$